

SECTION : A COMBINATION OF NEWTON METHOD AND SECANT METHOD FOR SOLVING NON LINEAR EQUATIONS

Nur Rokhman

Dept. of Computer Science and Electronics, Gadjah Mada University

Sekip Unit III, Bulaksumur Yogyakarta, 55281

Phone (0274) 546194

E-mail: nurrokhman@ugm.ac.id

ABSTRACT

Newton method is a famous method for solving non linear equations. However, this method has a limitation because it requires the derivative of the function to be solved. Secant method is more flexible. It uses an approximation value to the derivative value of the function to solved. Unfortunately, this method needs two initial values, compared to Newton method which only need one initial value. The use of numerical technique gives an approximation value of the solution. An initial value should be iterated until it approximates the solution. This means, the numerical solutions always contain error (ε). In this paper, the initial point of Newton method and are used to make the second initial value of the Secant method. Sention method is constructed by applying these initial values to the Secant method. The experiments show that Sention method only gives a small deviation, compared to Newton method.

Keyword: newton method; secant method; non linear equation

1. INTRODUCTION

The solution of an equation $f(x) = 0$ is an intersection point between the graph of $y = f(x)$ and the X axis. Assuming the initial value of the solution is x_0 , iteratively the solution could be found by using Equation (1) as illustrated in Figure 1 which is known as Newton method (Atkinson, 1985).

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (1)$$

The use of Newton method creates another problem. The derivation of some function is difficult. Secant Method solve this problem by using an approximation of the derivative value ($f'(x_n)$), that is the slope of a line connecting two points on the function. Secant method need two initial points. It is shown in Figure 2, by using two initial points x_0 and x_1 the solution could be found using Equation (2). (Atkinson, 1985).

$$x_{n+1} = x_n - f(x_n) \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \quad (2)$$

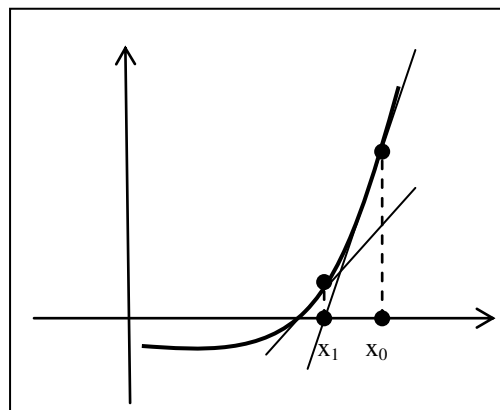


Figure 1. Newton method

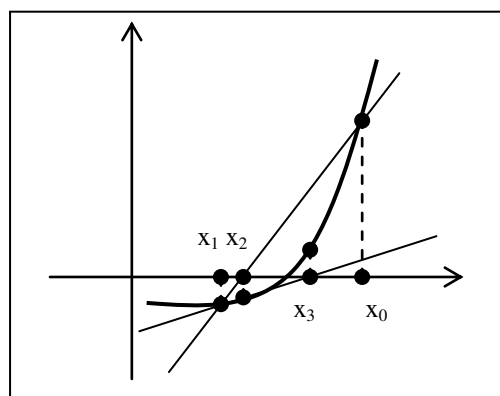


Figure 2. Secant method

2. SECTON METHOD

Numerical solution is found by iterating an initial value which moves to approximate the solution. The iteration process is stopped when the iteration reach the error tolerance (ε), otherwise no solution can be found.

Secton method is developed by using ε in the determination of derivative value of the function to be solved. A derivative of a function $f(x)$ is calculated using (Loomis, 1982) :

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (3)$$

If ε is very small, then derivative value of $f(x)$ at $x = x_n$ is

$$f'(x_n) = \frac{f(x_n + \varepsilon) - f(x_n)}{\varepsilon} \quad (4)$$

Applying Equation (4) into Equation (1) gives Secton method :

$$x_{n+1} = x_n - f(x_n) \frac{\varepsilon}{f(x_n + \varepsilon) - f(x_n)} \quad \square \square \square \square \quad (5)$$

Here is the algorithm of Secton method.

1. Determine ε
2. Determine the initial guess x_0
3. Do :

$$x_{n+1} = x_n - f(x_n) \frac{\varepsilon}{f(x_n + \varepsilon) - f(x_n)}$$

4. If $|x_{n+1} - x_n| > \varepsilon$ go to step 3

3. IMPLEMENTATION AND TESTING

The implementation of Secton method using Pascal Language can be seen in Figure 3. This function is tested to solve $x^6 - x - 1 = 0$ (Atkinson, 1985).

Table 1 shows the performance Newton method and Secton method using various error tolerance values. Table 2 shows the deviation result of Secton method compared to Newton method.

Some functions have unstable property. A small change on the input value makes a big difference on the output value. This condition is called *ill-conditioned* (Buchanan and Turner, 1992). The

Wilkinson polynomial is an example of this function.

$$w(x) = \prod_{i=1}^{20} (x - i) \quad (6)$$

This polynomial is very unstable. If the coefficient x^{19} is decreased by 2^{-23} , then the former result $w(20) = 0$ will be -6.25×10^{17} .

The root of the Wilkinson polynomial around $x = 20$ is 20.84691. Secton method works well in finding this root. Table 3 shows the performance of Secton method by using various kind of tolerance values. In this experiment, the initial guess is $x_0 = 21$.

Table 4 shows the ability of Secton method in finding all the real Wilkinson polynomial root by using all integer number between 1 and 21 as initial guesses.

```
function secton(eps,x0:real): real;
var x,xbaru : real;
begin
  xbaru := x0;
  repeat
    x := xbaru;
    xbaru := x - f(x*eps/f(x+eps) - f(x));
  until abs(x-xbaru) < eps;
  secton := xbaru;
end;
```

Figure 3. The implementation of Secton

4. CONCLUSION

The Secton method solves non linear equation by :

- a. Only one initial value
- b. No derivation of the function to be solved

The result of Secton method deviate less than 5% compared to Newton method. The Secton method can find all real root of the Wilkinson polynomial by less than 10 times of iteration.

5. FUTURE WORK

There still need more evaluation on the performance of the Secton method in many other problem such as : the determination of complex roots of the polynomial, the solution of system of non linear equation.

REFERENCE

- Atkinson, K., 1985, *Elementary Numerical Analysis*, John Wiley & Sons, Inc., New York.
- Buchanan, J.L. and Turner, P.R., 1992, *Numerical Methods and Analysis*, McGraw-Hill, New York.
- Loomis, L.H., 1982, *Calculus*, 3rd Edition, Addison-Wesley Publishing Company, Sydney.

Table 1. The performance comparison between newton method and section method for solving $x^6 - x - 1 = 0$

n	x_n					
	Newton method	Secton method				
		$\mathcal{E} = 0.1$	$\mathcal{E} = 0.01$	$\mathcal{E} = 0.001$	$\mathcal{E} = 0.0001$	$\mathcal{E} = 0.00001$
0	1.5	1.5	1.5	1.5	1.5	1.5
1	1.30049088	1.33182688	1.30386305	1.30083058	1.30052488	1.30049427
2	1.18148042	1.22117387	1.18550386	1.18188279	1.18152065	1.18148443
3	1.13945559	1.16358436	1.14127041	1.13962947	1.13947290	1.13945732
4	1.13477763	1.14219641	1.13497865	1.13479323	1.13477914	1.13477778
5	1.13472415	1.13641364	1.13473037	1.13472432	1.13472416	1.13472415
6	1.13472414	1.13509023	1.13472429	1.13472414	1.13472414	1.13472414

Table 2. The result deviation of Secton method compared to Newton method for solving $x^6 - x - 1 = 0$

n	$\mathcal{E} = 0.1$	$\mathcal{E} = 0.01$	$\mathcal{E} = 0.001$	$\mathcal{E} = 0.0001$	$\mathcal{E} = 0.00001$
0	0	0	0	0	0
1	2.41 %	0.26 %	0.03 %	0.00 %	0.00 %
2	3.36 %	0.34 %	0.03 %	0.00 %	0.00 %
3	2.12 %	0.16 %	0.02 %	0.00 %	0.00 %
4	0.65 %	0.02 %	0.00 %	0.00 %	0.00 %
5	0.15 %	0.00 %	0.00 %	0.00 %	0.00 %
6	0.03 %	0.00 %	0.00 %	0.00 %	0.00 %

Table 3. The performance of Secton method for solving Wilkinson polynomial around $x = 20$

n	$\mathcal{E} = 0.1$	$\mathcal{E} = 0.01$	$\mathcal{E} = 0.001$	$\mathcal{E} = 0.0001$	$\mathcal{E} = 0.00001$
1	20.90277	20.88554	20.88367	20.88348	20.88346
2	20.86153	20.85045	20.84955	20.84947	20.84946
3	20.84994	20.84700	20.84693	20.84692	20.84691
4	20.84748	20.84691	20.84691	20.84691	
5	20.84702				
6	20.84693				
7	20.84691				

Table 4. Various initial guesses and solution of the Wilkinson polynomial by using Secton method

x_0	solution	x_0	solution
1	1.00000	11	8.91725
2	2.00000	12	8.91725
3	3.00000	13	1.00000
4	4.00000	14	8.91725
5	5.00000	15	8.00727
6	6.00001	16	8.91725
7	6.99970	17	8.00727
8	8.00727	18	8.91725
9	8.91725	19	8.91725
10	8.91725	20	8.00727
		21	20.84691